RESULTS OF AN INVESTIGATION INTO NON-ISOTHERMAL FLOW OF AN INCOMPRESSIBLE LIQUID IN UNDERGROUND CHANNELS AND PIPES

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Dimensionless functions have been obtained which make it possible to determine the temperature of an incompressible liquid, moving in an underground channel or pipe, for two cases of practical importance: when the temperature of the liquid in the initial section of channel or pipe is constant, and when this temperature varies according to a periodic law.

An analytical solution of the problem of the change in temperature of an incompressible liquid moving inside an underground channel or pipe, is fairly complex. The complexity is caused by the fact that the stream of liquid moving in an underground channel, having a temperature differing from that of the surrounding rock or earth, is constantly changing the temperature field in the material around the channel.

Consequently the conditions for heat transfer from the liquid to the surrounding earth is constantly changing. When the temperature (t_0) of the liquid in the initial section ($x = 0$) of the channel is constant, technicians in a number of fields apply an extremely approximate solution of the problem, in which a constant average value is taken for the heat transfer coefficient k. The required temperature $t(x)$, at a distance x from the initial section, is found from the expression

$$\frac{t(x)-\vartheta_e}{t_0-\vartheta_e}=\exp\left[-\frac{k\pi d_0}{cG}x\right]. \qquad (1)$$

In the majority of cases of practical interest it is impossible to take k as a constant with time in Eq. (1), and therefore the solution becomes unacceptable.

More accurate solutions of this problem are known which are obtained under the following assumptions:

a) the temperature of the liquid in any given cross section of the channel is taken to be the same over the whole area;

b) the coefficient for heat exchange between the liquid and the internal surface of the channel is taken to be constant along the length of the channel and in time;

c) the heat flow into the surrounding earth along the axis of the channel is small compared with the heat flow perpendicular to the axis, and can be neglected (consequently $\partial\vartheta/\partial x = 0$);

d) the thermophysical properties of the material protecting the channel and of the surrounding earth, are the same.

Van Heerden [1], in connection with a study of the air cooling of carbon layers, succeeded in obtaining an equation for the temperature of the air moving in the channel under conditions where the temperature is constant with time ($t_0 = \text{const}$) in the initial section:

$$t(x^*, \tau^*) = t_0\varphi(x^*, \tau^*), \qquad (2)$$

$$\varphi(x^*, \tau^*) = 1-\frac{2}{\pi}\int_0^\infty \frac{\exp(-\tau^*\mu^2)}{\mu}\times$$

$$\times\sin\left\{x^*\frac{\dfrac{2\,\mathrm{Bi}}{\pi\mu^2}}{\left[\dfrac{\mathrm{Bi}}{\mu}I_0(\mu)+I_1(\mu)\right]^2+\left[\dfrac{\mathrm{Bi}}{\mu}Y_0(\mu)+Y_1(\mu)\right]^2}\right\}\times$$

$$\times\exp\left\{-x^*\frac{I_1(\mu)\left[\dfrac{\mathrm{Bi}}{\mu}I_0(\mu)+I_1(\mu)\right]+Y_1(\mu)\left[\dfrac{\mathrm{Bi}}{\mu}Y_0(\mu)+Y_1(\mu)\right]}{\left[\dfrac{\mathrm{Bi}}{\mu}I_0(\mu)+I_1(\mu)\right]^2+\left[\dfrac{\mathrm{Bi}}{\mu}Y_0(\mu)+Y_1(\mu)\right]^2}\right\}d\mu. \qquad (3)$$

A Comparison of Temperatures Calculated Analytically by Eq. (2) and Experimentally by Eqs. (11)–(13)

Time after start of operation; hours	t in °C at distances x, in meters, from the initial section							
	35			75			200	
	analyt.	experiment		analyt.	experiment		analyt.	experiment
	Eq. (2)	Eq. (11)	Eq. (12)	Eq. (2)	Eq. (11)	Eq. (13)	Eq. (2)	Eq. (11)
52	26.6	24.3	24.4	23.6	21.2	21.2	17.5	17.5
2600	28.8	27.5	27.6	27.5	26.4	26.4	23.8	23.8
5200	29.0	27.9	28.0	28.0	26.9	26.9	24.7	24.7
52000	29.6	28.8	28.8	29.1	28.2	28.2	26.8	26.8

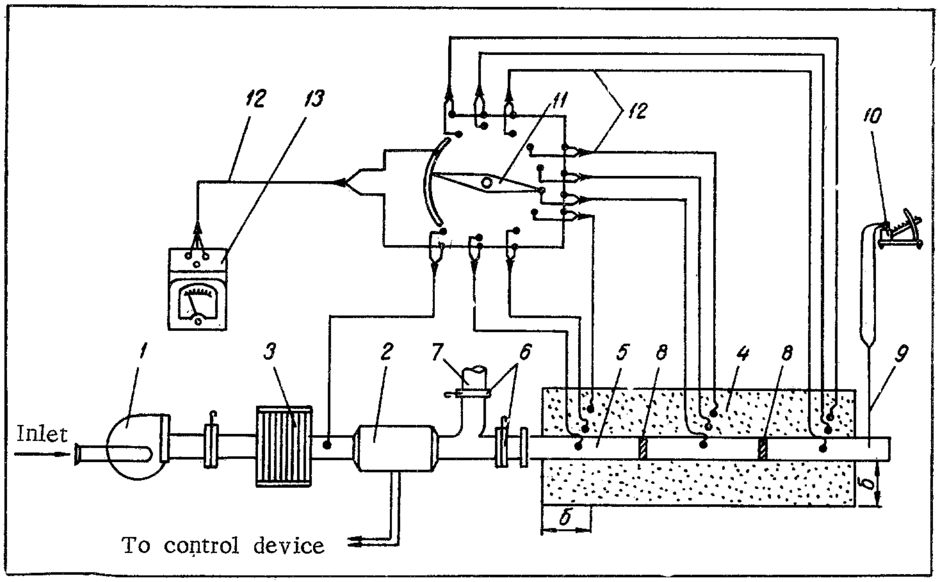

Fig. 1. Diagram of the experimental equipment: 1) centrifugal blower; 2) electrical air heater; 3) refrigerator evaporator; 4) layer of material simulating a large mass (earth, rock); 5) steel pipe; 6) gates; 7) branch required during warmup of the electrical air heater; 8) thermal isolation insertions for decreasing the effect of heat conduction along the pipe; 9) pneumometric tube; 10) micromanometer; 11) 12 contact switch. 12) connecting wires for joining thermistors; 13) unbalanced measuring bridge.

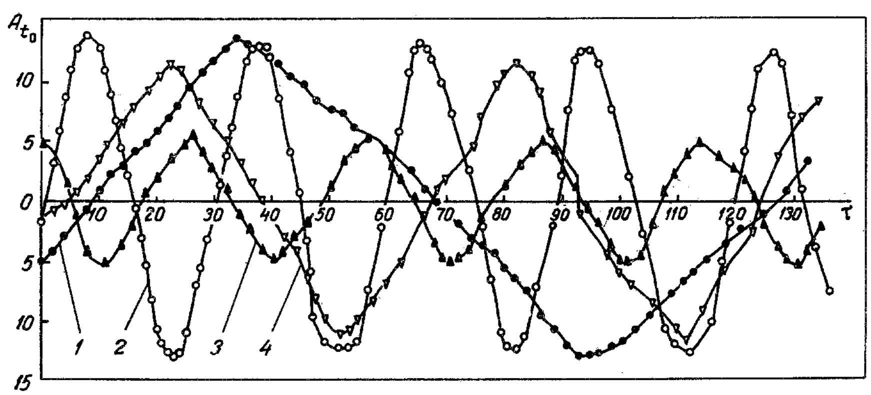

Fig. 2. Graph of the air temperature in the initial pipe section for several experiments in the second part of the investigation (A_{t_0} in °C, τ in min): 1) No. 10 (T_0 = 2 hr, A_{t_0} = 13.6°; w = 15 m/sec); 2) No. 4 (T_0 = 0.5 hr, A_{t_0} = 13.4°; w = 8.5 m/sec); 3) No. 6 (T_0 = 0.5 hr; A_{t_0} = 5°; w = 36 m/sec); 4) No. 11 (T_0 = 1 hr; A_{t_0} = 11.4°; w = 15 m/sec).

In collaboration with O. A. Berezin we have obtained [2] a solution for the case where, in the initial section of the channel, there is an incompressible liquid having a temperature which varies periodically as $t(0, \tau) = t_0 \cos \omega\tau$:

$$t(x, \tau) = t_0 \exp(-Mx)\cos(\omega\tau - Nx). \qquad (4)$$

Here

$$M = \frac{h}{w}\frac{\beta}{\mathrm{Bi}} \times \left[\frac{\beta}{\mathrm{Bi}}[\mathrm{Ker}'^2(\beta) + \mathrm{Kei}'^2(\beta)] - \right.$$
$$\left. - [\mathrm{Ker}(\beta)\mathrm{Ker}'(\beta) + \mathrm{Kei}'(\beta)\mathrm{Kei}(\beta)]\right] \times$$
$$\times \left[\left[-\mathrm{Ker}(\beta) + \frac{\beta}{\mathrm{Bi}}\mathrm{Ker}'(\beta)\right]^2 + \right.$$
$$\left. + \left[-\mathrm{Kei}(\beta) + \frac{\beta}{\mathrm{Bi}}\mathrm{Kei}'(\beta)\right]^2\right]^{-1}, \qquad (5)$$

$$N = \frac{\omega}{w} + \frac{h}{w}\frac{\beta}{\mathrm{Bi}} \times$$
$$\times \left[\mathrm{Kei}(\beta)\mathrm{Ker}'(\beta) - \mathrm{Ker}(\beta)\mathrm{Kei}'(\beta)\right] \times$$
$$\times \left[\left[-\mathrm{Ker}(\beta) + \frac{\beta}{\mathrm{Bi}}\mathrm{Ker}'(\beta)\right]^2 + \right.$$
$$\left. + \left[-\mathrm{Kei}(\beta) + \frac{\beta}{\mathrm{Bi}}\mathrm{Kei}'(\beta)\right]^2\right]^{-1}. \qquad (6)$$

Numerical values of the Thomson functions Ker, Ker', Kei, Kei', can be found in [3].

The analytical equations (2)–(6) are extremely cumbersome and inconvenient for practical use. In addition a number of assumptions are made in deriving these equations which obscure the physical picture of the phenomena. Of these assumptions the crudest is the third, which assumes that there is no heat flow into the surrounding earth along the axis of the channel.

This assumption is least acceptable for short channels and for short time intervals during which the channels are used, when the temperature field in the earth along the axis is most nonuniform. In order to take into account this factor, as well as to obtain simpler equations convenient for practical calculations, we have carried out experimental investigations on equipment simulating an underground channel (Fig. 1).

Atmospheric air was drawn through blower 1 and, after heating in the specially constructed electrical air heater 2, was passed into pipe 5, surrounded by a layer of material 5 simulating a large mass (rock, earth). In order to make possible wider variations in the air temperature, the evaporator of a refrigerating unit was included, enabling the air to be cooled when required. All the remaining details of the equipment are clear from the figure.

The experimental investigations fell into two sections: in the first part we investigated the behavior on passing air at constant temperature into the initial section, while in the second part we studied the processes which developed on admitting air into the channel at a temperature which varied periodically with time. To guarantee the required changes in the temperature of the air entering the pipe, provision was made for automatic control of the sections of the electrical air heater (electric ovens) according to a preset program. To accomplish this the ovens were connected to a control device (electropneumatic controller type KEP-12U, manufactured by the Kaluga Pyrometric Equipment Factory). Using the control device it was possible to vary over wide limits both the amplitude and the period of oscillation of the air temperature after the electrical air heater.

Extremely important factors here are the choice of material simulating a large mass of earth, and the choice of the dimensions necessary in the model. Experiments were carried out with various materials differing significantly in their thermophysical characteristics, namely sawdust, microporous rubber, and sand. It turned out that the most reliable and steady results were obtained using dry sand, which was chosen as the basic material for filling the housing around the pipe.

The thickness of the sand layer should simulate the boundary conditions characteristic of the placement of a channel in an infinite mass. The thickness of sand required is found most simply by experimental means. We assumed that it was permissible to carry out an experiment of the first kind after passing air at constant temperature through the pipe until the temperature through the pipe until the temperature at the surface of the sand around the initial section remained almost constant. It turned out that for the performance of a lengthy experiment (6–8 hours) a filling thickness of about two pipe diameters was sufficient. For increased confidence we have taken a thickness $\delta = 2.5d_0$. To eliminate the effects of conditions at the vertical boundary of the layer of material, the initial and final sections were located at the same distance $2.5d_0$ from the end wall of the housing. It can be shown that the filling thickness taken is completely satisfactory for carrying out the second series of experiments involving periodic changes in the air temperature. To do this we use the well known equation for determining the damping of temperature oscillations in an infinite mass [4], allowing for the characteristics of heat inflow from a cylindrical cavity,

$$\delta = \frac{r_0}{r_0 + \delta}\sqrt{2}\sqrt{\frac{a}{\omega}}\ln n. \qquad (7)$$

Then for $T_0 = 1$ hour, $n = 10^{10}$; for $T_0 = 12$ hours, $n > 10^3$.

Thus the conditions were fulfilled which made it possible to calculate that the equipment simulated a channel or pipe located in an infinite mass.

In conformity with the theory of thermal conductivity [5, 6], the similarity of the processes for the first set of experiments (air temperature in the initial section constant with time) is guaranteed by maintaining constant values for the two criteria: Fo (Fourier) and Bi (Biot), and for the second set (the air temperature in the initial section changes periodically) by the constancy of the criteria: Pd (Predvoditelev) and Bi. Here $\mathrm{Fo} = a\tau/r_0^2$; $\mathrm{Bi} = r_0\alpha/\lambda$, $\mathrm{Pd} = \omega r_0^2/a$.

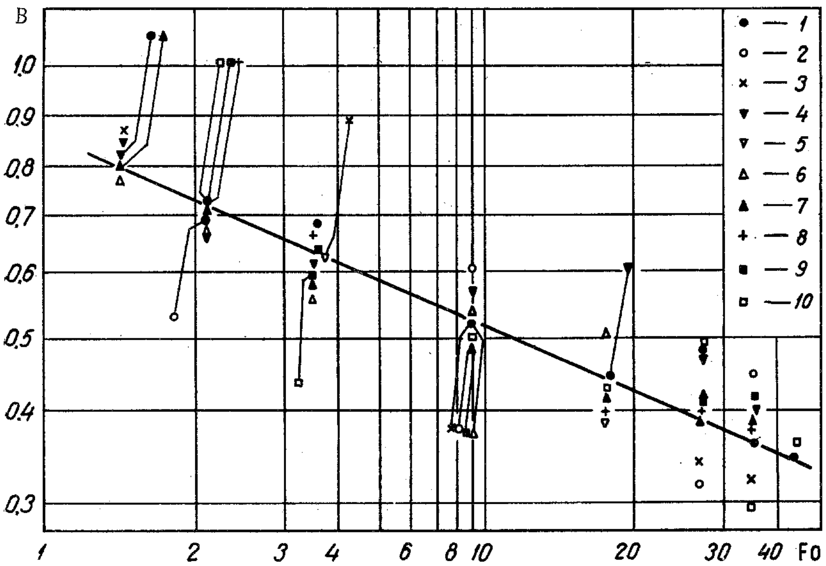

Fig. 3. Results of analyses of the experiments to determine the form of the equation for ψ (normalized to $x/d_0 = 50$; $B \equiv \psi\, Bi^{0.8}(x/d_0)^{0.4}$): 1) Bi = 1.45, $x/d_0 = 50$; 2) Bi = 1.45, $x/d_0 = 100$; 3) Bi = 1.45, $x/d_0 = 35$; 4) Bi = 0.75, $x/d_0 = 50$; 5) Bi = 0.75, $x/d_0 = 35$; 6) Bi = 1.2, $x/d_0 = 50$; 7) Bi = 1.2, $x/d_0 = 100$; 8) Bi = 1.5, $x/d_0 = 50$; 9) Bi = 1.39, $x/d_0 = 100$.

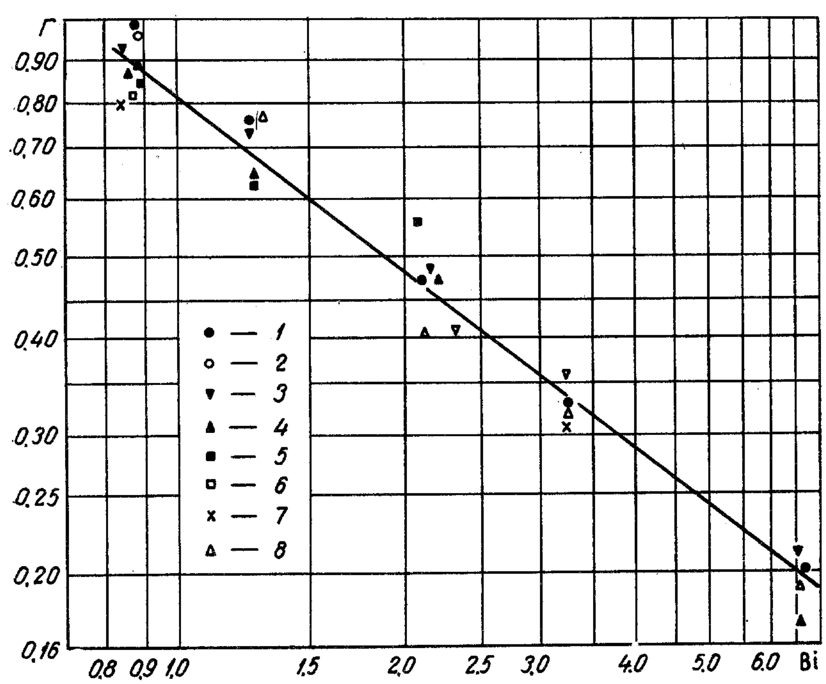

Fig. 4. Results of analyses of the experiments to determine the form of the equation for ψ' (normalized to $x/d_0 = 50$; $\Gamma \equiv \psi'/\beta^{0.25}(x/d_0)^{0.15}$): 1) $\beta = 3.31$, $x/d_0 = 100$; 2) $\beta = 2.9$, $x/d_0 = 50$; 3) $\beta = 0.875$, $x/d_0 = 50$; 4) $\beta = 0.62$, $x/d_0 = 100$; 5) $\beta = 1.24$, $x/d_0 = 100$; 6) $\beta = 1.66$, $x/d_0 = 50$; 7) $\beta = 1.24$, $x/d_0 = 35$; 8) $\beta = 3.31$, $x/d_0 = 35$.

The experiments were carried out using either a steel pipe of 0.08 m diameter and wall thickness 3.00 mm, or pipe made from thin sheet steel with a diameter of 0.03 m and wall thickness 0.5 mm. In order to eliminate the effects of the thermal conductivity of the steel (along the axis) in the 0.08 m diameter pipe, it was first cut into sections, between which were inserted thermal insulators in the form of ring gaskets.

The following temperatures were measured during the experiments:

The temperature of the air entering the electrical air heater;

The temperature of the air at two points after heating: in the middle of the first pipe section (during the experiment), and in the branch pipe for venting the air into the atmosphere during warm-up of the air heater (during the establishment of stationary conditions for the process);

The temperature of the air at several points along the pipe (in different experiments the number of points varied, but was usually 4 or 5);

The temperature of the sand in the three sections at various distances from the pipe;

The ambient temperature.

The main temperature measurements were made using semiconductor thermistors connected via switch 11 to the unbalanced bridge 13 (Fig. 1); these measurements were accurate to 0.1°. The time lag of the thermistors did not exceed 50 sec. In view of the temperature oscillation periods used in the experiments (not less than 0.5 hour), this time lag is considered entirely satisfactory.

The air temperatures in the laboratory and branch pipe were measured with mercury thermometers with scales graduated in 0.1° units. The rate of air movement was regulated using the valves (6) and measured by means of a pneumometric tube connected to micromanometer 10.

In the first series, experiments were made to distinguish the rate of air movement in the pipe w, the pipe diameter d_0, and the air temperature at the entrance to the pipe t_0, keeping the latter constant during each experiment.

The second series of experiments involved changing the temperature of the air entering the channel according to a periodic time-dependent function, i.e., $t_0 = A_{t_0} \sin \omega\tau$. By switching in the control device in the feed supply for the air heater, different amplitudes and oscillation frequencies of the temperature were obtained in the various experiments. These are illustrated in Fig. 2.

In treating the data obtained in the experiments we have assumed that the temperature change in the moving air, for the case where the input air temperature t_0 = const (first series of experiments), can be described by the equation

$$\frac{t(x,\ \tau)-\vartheta_e}{t_0-\vartheta_e}=\exp\left(-\frac{h}{w}\psi x\right). \qquad (8)$$

Equation (8) can be written in the form

$$\frac{t(x,\ \tau)-\vartheta_e}{t_0-\vartheta_e}=\exp\left(-\frac{k'S}{\gamma wcF}x\right). \qquad (9)$$

Thus the object of the first part of the investigation was to find a dimensionless expression for the thermal diffusion coefficient under conditions where air enters the first section of the channel at constant temperature. This coefficient is a function of the thermophysical properties of the surrounding mass and of the moving medium, of time, and of the heat transfer conditions at the boundaries of the system. In like processes (Bi = idem, Fo = idem) the diffusion coefficient has the same value. Therefore, in treating the experiments in the first part of the investigation it was necessary to find a functional dependence of the form

$$\psi=f(\mathrm{Bi},\ \mathrm{Fo},\ x/d_0). \qquad (10)$$

From the experiments comprising the first part of the investigation we obtained the graph shown in Fig. 3, from which the following expression is obtained:

$$\psi=4.1\,\mathrm{Fo}^{-0.23}\mathrm{Bi}^{-0.8}(x/d_0)^{-0.4}. \qquad (11)$$

For convenience, two approximate working formulas are proposed:

for $0 \leqslant x/d_0 \leqslant 50$

$$\psi=0.95\,\mathrm{Fo}^{-0.25}\mathrm{Bi}^{-0.8}, \qquad (12)$$

for $50 \leqslant x/d_0 \leqslant 100$

$$\psi=0.7\,\mathrm{Fo}^{-0.25}\mathrm{Bi}^{-0.8}. \qquad (13)$$

There is one question which has yet to be considered. When Fo is small, the coefficient ψ is very close to unity. As follows from our experiments, confirmed in subsequent field investigations, for Fo less than some limiting value ($\mathrm{Fo} \le \mathrm{Fo}_{\lim}$) coefficient ψ can be taken as unity. In conformity with Eqs. (12) and (13) these limiting values must satisfy the following equations:

for $0 \leqslant x/d_0 \leqslant 50$

$$\mathrm{Fo}_{\lim}^{0.25}\mathrm{Bi}^{0.8}=0.95, \qquad (14)$$

for $50 \leqslant x/d_0 \leqslant 100$

$$\mathrm{Fo}_{\lim}^{0.25}\mathrm{Bi}^{0.8}=0.7. \qquad (15)$$

It is of interest to compare the results of calculations using Eqs. (11)–(13) with the analytical solution (2) obtained by K. Van Heerden. This solution was obtained for extremely long underground channels in use for quite a long time. Therefore the assumption that there is no heat flow along the axis of the channel ($d\vartheta/dx = 0$) is not a crude one. In other cases this assumption may distort the real picture. In the table a comparison is made of the results of calculating the temperatures for various lengths of time after the start of operation of an underground channel having a diameter of 1.0 m, if the temperature of the environment $\vartheta_e = 10°$ C and the air temperature in the initial section is $t_0 = 30°$ C. The air flow rate is 8000 m^3/hr, and the thermophysical constants of the environment (granite) are: $\lambda = 3.48$ W/m · deg; $\gamma = 2800$ kg/m^3; c = 0.93 kJ/kg · deg; $a = 4.85 \cdot 10^{-3}$ m^2/hr. Under these conditions Bi = 1.0; Fo = 0.0194τ.

The results in the table on p. 236 show that for short underground channels calculations based on Eq. (2) are not sufficiently accurate and the experimental equations (11)–(13) should be recommended, these allowing changes in Bi and Fo over extremely wide limits and restricted only by the conditions (14) and (15).

For the second series of experiments ($t_0 = At_0 \sin \omega\tau$) we thought it advisable to keep the working formula in a universal form, in accordance with which the amplitude of the air temperature oscillation at a distance x from the initial section is found from the expression

$$\frac{A_{t_x}}{A_{t_0}} = \exp\left(-\frac{k''S}{\gamma wcF}\, x\right). \tag{16}$$

The quantity ψ' is a dimensionless constant, taking into account the diffusion of heat into the interior of the environment, and is a function of the two generalized variables Bi and Pd. Similarity of processes is assured when Bi = idem and Pd = idem. Therefore, in working out the experiments in the second series we tried to find a functional equation in the form

$$\psi' = f(\mathrm{Bi},\ \mathrm{Pd},\ x/d_0). \tag{17}$$

It turned out to be more satisfactory to take $\sqrt{\mathrm{Pd}} = \beta = \sqrt{\omega/a r_0}$ instead of Pd = $\omega r_0^2/a$.

From the second series of experiments the graph shown in Fig. 4 was obtained, on the basis of which the required equation can be written in the form

$$\psi' = 1.43\,\mathrm{Bi}^{-0.73}\,\beta^{0.25}\,(x/d_0)^{-0.15}. \tag{18}$$

For relative distances in the limits $0 \le x/d_0 \le 100$ the simpler equation

$$\psi' = 0.8\,\mathrm{Bi}^{-0.73}\,\beta^{0.25} \tag{19}$$

is proposed.

As in the case of a liquid at constant temperature entering the channel, there are practical limits to the values of the criterion $\beta = \sqrt{\mathrm{Pd}}$ which, when exceeded, means that coefficient ψ' can be taken as unity. According to Eq. (19) this limiting value can be found from the equality

$$\beta_{\lim}^{0.25} = 1.25\,\mathrm{Bi}^{0.73}. \tag{20}$$

The physical meaning of this relationship is obvious: since $\beta = \sqrt{\omega/a r_0}$, then for oscillations of a frequency equal to or greater than those obtained from Eq. (20) there will be such a large dissipation of heat into the earth that all the heat supplied to the surface of the channel, and determined by coefficient α, will be absorbed.

Comparison of the results of calculations using the proposed formulas (18) and (19) and the analytical equation (4), showed a very good agreement. Just as in the previous case, as a consequence of the assumption $d\vartheta/dx = 0$, the amounts by which the temperature oscillations were damped were decreased when calculated analytically. Therefore in this case also it can be recommended that engineering calculations involving nonisothermal flow of a noncompressible liquid in underground channels and pipelines be based on the quite simple experimental equations which we have obtained.

NOTATION

$x^* = \alpha S/Q_{sp}$; S) perimeter or internal surface, per unit length of channel; Q_{sp}) specific amount of heat carried by the liquid through the channel cross section per unit time; $\tau^* = a\tau'/\tau_0^2$) modified Fourier criterion ($\tau' = \tau - x/w$); τ) time; I_0, I_1, Y_0, Y_1) Bessel functions of the first and second kinds of a real argument of the zero and first orders; $\mu = \sqrt{s/a\tau}$; s) an operator defined by its modulus $s = |s|\exp(i\varphi)$; φ) angle read off counterclockwise from the real semiaxis; Bi = $= \alpha r_0/\lambda$) Biot's criterion; $\beta = \sqrt{\omega/a r_0}$; α) heat transfer coefficient from the moving air to the internal surface of the channel; λ) thermal conductivity coefficient of the earth; $\vartheta(r, \tau)$) earth temperature ($\partial\vartheta/\partial x = 0$); a) thermal diffusivity coefficient of the earth; $t(x, \tau)$) temperature of the liquid moving in the channel; $h = \alpha Sw/cG$; G) amount of liquid transported per unit time; w) rate of movement of liquid in the channel; n) multiplicity of the decrease in amplitude of the temperature oscillation at distance δ from the surface of the pipe; $\omega = 2\pi/T_0$) angular frequency of the temperature oscillation; r_0 is the internal radius of the channel (or equivalent radius); ψ, ψ') thermal propagation coefficients, functions of the boundary conditions and the thermophysical and time characteristics of the processes; c, γ) specific heat and density of the liquids; ϑ_e) initial temperature of the environment (kept invariable at a sufficient distance); F) cross-sectional area of the channel; $k' = \alpha\psi$) nonstationary heat exchange coefficient between the moving liquid and the environment, with t_0 = const; $k'' = \alpha\psi'$) the same, but with periodic changes in the temperature of the liquid.

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3 May 1966

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